

Fairfax County Public Schools Socio-Economic Tipping Point Study of Elementary Schools

Recently, Joseph Murphy, a nationally recognized expert on educational improvement, suggested to Fairfax County Public Schools' (FCPS) Leadership Team that one of the ways to close achievement gaps is for students living in poverty to attend schools with lower proportions of poverty. His suggestion stemmed from studies that found students living in poverty do better when attending low-poverty schools than schools with high concentrations of poverty. That is, the impact of a student's individual poverty can be ameliorated to some extent by enrollment in schools with low concentrations of poverty. This paper describes how one factor, *school poverty*, was investigated to determine an empirically-based approach for giving all FCPS schools a better chance of attaining overall school success and to give all FCPS students a better chance of achieving individual success. The report describes the development of a statistical model useful in guiding FCPS' School Board and Leadership Team in decisions about how to help struggling schools.

Background

No single variable explains overall school success in producing high academic performance among its students. In fact, education research provides strong evidence that a variety of factors, often in concert, impact schools. Among these factors are teacher skills and instructional quality, student attendance, (Graber, 2009), expectations of parents and teachers, previous student retention, class size and school size ("Teacher Quality," 2005; "Primer: Education," 2013). However, even with the understanding that multiple factors impact schools, the pursuit of many different factors by a school can have the negative impacts of:

- Reducing a school's ability to focus its improvement efforts on a reasonable number of issues;
- Spread financial and human resources too thinly to fully accomplish strategies; and
- Overwhelm school staff and students with the collection and monitoring evidence of success.

As schools, especially those struggling academically, seek to improve the outcomes for students, they may be best served by prioritizing and addressing the most critical factors influencing high quality teaching and learning. That is, schools should consider strategies which address the factors that:

- Are known to be high in their influence on the teaching and learning process;
- Are under considerable control or influence of the school or the school division; and
- Avoid adding more negative impacts to the school.

Poverty is well documented as one of the most influential factors affecting student achievement. And, poverty can be defined in two different ways: (a) *individual student poverty* is the eligibility of a single student for free and reduced meals; (b) *school poverty* is more cumulative in nature because it reflects the overall percentage of students at the school who are eligible for free and reduced meals. While student poverty and school poverty are related factors, the distinction between them is a critical one. Having only a few impoverished students presents instructional challenges for a school, but having great numbers of impoverished students multiplies that challenge such that some schools find meeting all of their academic performance goals unrealistic and demoralizing.

Historically, the response has been to provide additional resources to individual schools based on the percentage of impoverished students at the school. The intent has been to lessen the challenge of high numbers of impoverished students. However, research has shown that high-poverty schools tend to

exhibit differences (versus low-poverty schools) that go beyond the aggregate level of individual student poverty. These differences (e.g., lower rates of teacher retention, more novice teachers, lower parent involvement, etc.) serve to act as counterweights to any resources channeled to these schools. Thus, the resources may help schools mitigate and level off the effects of poverty but do not counter poverty sufficiently to raise achievement at FCPS' most impoverished schools. And, while there have been cases locally and nationally of schoolwide success with high percentages of impoverished students who receive additional resources, the more common outcome has been schools still struggle to meet state and local schoolwide performance expectations. For example, in FCPS, we have outliers that are succeeding even with high levels of school poverty, but they tend to be the exceptions. (See Appendix A for additional research on school poverty.)

The Department of Professional Learning and Accountability's Office of Program Evaluation (OPE) conducted a study to better understand the connection between overall school poverty and overall school academic performance. This study examined the merits of capping the percentage of overall school poverty at any one school. The goal of the study was to find empirical evidence of a "tipping point" for elementary school poverty in FCPS (i.e., a level of school poverty at which success is statistically unlikely). Underpinning this strategy is the intent to reduce a school's challenges associated with educating a high percentage of low-income students. Although the needs of individual students from impoverished backgrounds do not change simply because there are fewer of them at a single school, the increased capacity of a school to address their needs seems supported by evidence presented in this paper. That is, is there a point (percentage) at which there are so many impoverished students at a single school that, even with additional division resources, it is unlikely the school will consistently meet academic expectations? If such a point can be identified, it can serve as a catalyst in discussions about alternative approaches for giving all schools the best chance of being viewed as successful. Elementary schools were the focus of this study because *school* was the primary unit of analyses and there are more elementary schools with more varied levels of poverty than middle or high schools in FCPS. Moreover, poverty data are generally more accurate at the elementary level than at the middle school or high school level. (Additional discussion of tipping points and how they have been studied is available as Appendix B).

Results from the Socio-Economic Tipping Point Study of Elementary Schools

To investigate possible tipping points in FCPS elementary schools, the OPE considered two research questions:

1. *What, if any, level of school poverty is associated with FCPS schools' capacity to meet academic performance expectations?*
2. *What is the typical impact on students attending a higher poverty FCPS school?*

Study Question 1: What, if any, level of school poverty is associated with FCPS schools' capacity to meet academic performance expectations?

The basic concern behind this question was whether reaching a certain concentration of poverty greatly reduced the likelihood of student success. For example, if most every FCPS school with poverty levels above 50 percent were unable to meet the state-defined levels of passing the SOL tests, then FCPS would do well to treat schools above the 50 percent tipping point differently than other schools. Underlying this concern is the notion that at a certain point, referred to here as the tipping point, schools are either overly burdened or spread too thin in attending to the needs of students to be successful.

Summary Finding: *The study took both a graphical and a statistical approach to answering this question using SY 2011-12 poverty and student achievement data.¹ Graphs of school-level pass rates (i.e., the percentage of students in the school above benchmarks on the reading or mathematics SOL tests) and school poverty indicated in general that as levels of school poverty increased, schools were less likely to meet academic performance expectations (i.e., schools were more likely to have SOL pass rates falling below expected levels).*

And, almost all schools with poverty levels of 45 percent or higher were unable to reach expected pass rate levels in reading or math. Follow-up statistical analyses found statistical evidence that two tipping points exist in FCPS. The reading data provided the most consistent findings as it indicated two tipping points occurring at 20 and 40-45 percent school-level poverty. Thus, FCPS schools with greater than 20 percent poverty are much less likely to meet performance expectations than those with less than 20 percent poverty. And, once poverty levels at a school reach 40 percent or more, FCPS schools are unlikely to meet expectations for school performance.

Graphical Examination of Data

Figure 1 depicts the relation between reading pass rates for schools (on the vertical axis) and the overall percentage of poverty at a school (on the horizontal axis). Each diamond in the graph represents one FCPS elementary school. The red line at 90 percent helps identify (as does the gray shaded area above the line) those schools meeting performance expectations (i.e., with an overall reading SOL pass rate above 90 percent; this pass rate represents both FCPS' locally identified goal, as well as the Virginia Department of Education Annual Measurable Objective (AMO) in reading, rounded up to the nearest ten).

During SY 2011-12, the school poverty in FCPS elementary schools ranged from approximately 0 to 80 percent, with a mean of 26 percent. Figure 1 shows an overall negative association between school poverty and pass rates, meaning that as school poverty levels increased, pass rates decreased. Dividing school poverty into three zones on this graphic revealed that almost all FCPS elementary schools with poverty levels below 20 percent (no shading at the left side of the figure) had more than 90 percent of their students pass the reading SOL. Moving to the right on the school poverty percentage, Zone II (shaded in light yellow), which depicts school poverty levels from 20 to 45 percent, shows that the majority of schools still had more than 90 percent of their students pass the reading SOL, but quite a few schools fell below this desired level. Lastly, in Zone III, which depicts school poverty levels above 45 percent, most of the schools did not have 90 percent or more of their students pass the reading SOL. The eight schools above the red line in Zone III (of 28 total schools) would be those that are succeeding even with high levels of school poverty.

Similarly, Figure 2 depicts the overall negative association between school poverty (horizontal axis) and mathematics pass rates for schools (on the vertical axis) and the overall percentage of poverty at a school (on the horizontal access). The red line at 70 percent and the gray overlay above it identifies schools with an overall mathematics SOL pass rate above 70 percent (this pass rate represents the Virginia Department of Education AMO in mathematics for SY 2011-12, rounded up to the nearest ten). Once again, dividing school poverty into three zones indicates that almost all of the schools with poverty levels below 20 percent were above the SY 2011-12 benchmark (Zone I), those between 20 and 45 percent poverty were likely to be above the benchmark but quite a few schools fell below it (Zone II), and among schools with more than 45 percent poverty, few typically exceeded the desired 70 percent pass rate benchmark (Zone III). The three schools above the red line (of 28 total schools in Zone III) are those demonstrating success in mathematics even with high levels of school poverty.

Figure 1: Scatter Plot of School Reading Pass Rate and School Poverty in SY 2011-12

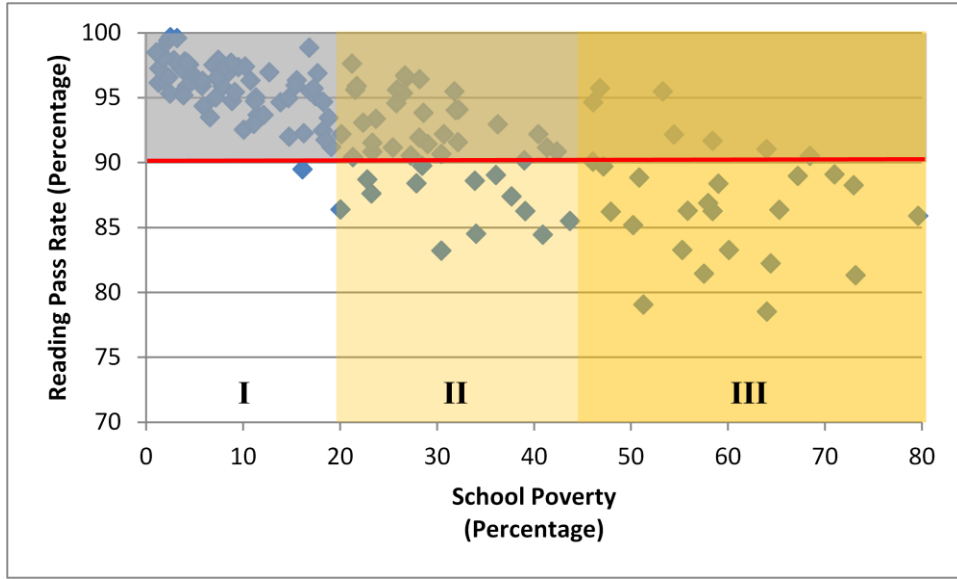
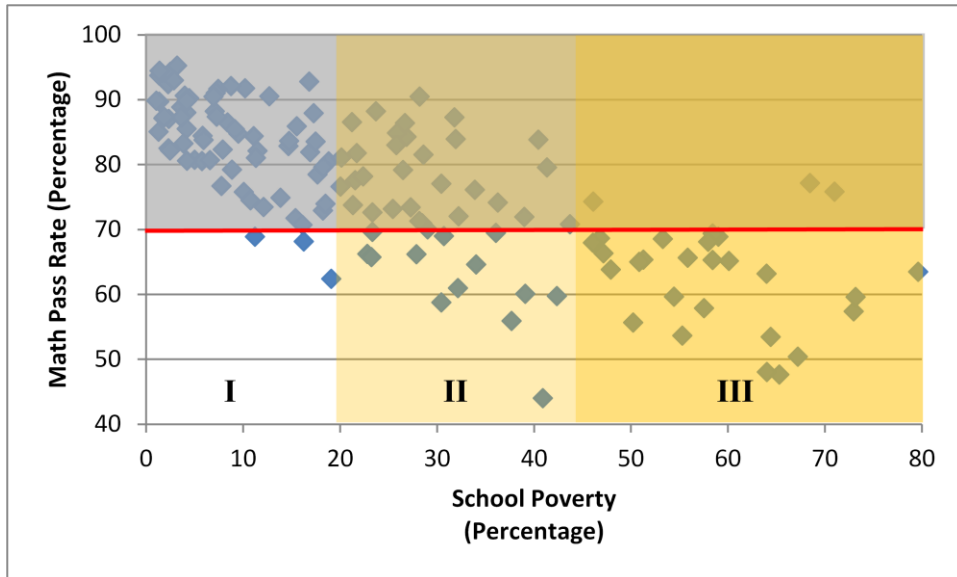


Figure 2: Scatter Plot of School Mathematics Pass Rate and School Poverty in SY 2011-12



Statistical Analyses

As described above, this study was in search of a critical level of school poverty (“tipping point”) at which unprecedented change occurs. Regression analyses revealed the possibility of two significant tipping points in the reading pass rate data, and additional multilevel analyses pinpointed the levels at 20 ($p < .05$), and 40-45 percent school poverty ($p < .001$). For mathematics achievement, results did not yield a consistent or clear pattern between school poverty and performance. The remainder of this paper focuses on reading performance since school poverty demonstrated a consistent link to it. [For additional details on the regression and hierarchical linear modeling analyses, see Appendix C.]

Study Question 2: What is the typical impact on students attending a higher poverty FCPS school?

Having established the existence of tipping points, this question now turns to what the tipping points signify with respect to how individual students achieve within the schools. Behind this study question is the notion that FCPS would like to ensure that our students have the same opportunity to achieve, regardless of which schools they attend. Or, stated another way, analyses for this question sought answers to whether a student attending one FCPS elementary school with high poverty would have had a different level of achievement had that same student attended an elementary school with low poverty. And, if so, what would be different about that student's achievement in the schools with the different levels of poverty? For example, do poor children attending schools with overall lower-poverty levels outperform their counterparts at higher-poverty level schools?

Summary Finding: *Analyses that allowed school poverty to be teased apart from individual student poverty revealed that school poverty at the elementary level had a demonstrably separate, though smaller, negative association with student learning than individual poverty: school poverty was associated with an average decrease of 8 to 18 scale score points on the SOL reading test. The tipping point identified at 20 percent poverty reflected an acceleration of the overall downward trend in student scores equal to an additional 7-point decrease. The 20 percent tipping point indicates that schools with poverty levels above 20 percent were not as successful with students as those below the 20 percent poverty line. The tipping points identified at 40 and 45 percent poverty reflected the reverse, a slowing down of the observed downward trend, equal to approximately 13 to 16 points. These latter tipping points do not mean that schools above 40 or 45 percent poverty had students with higher test scores; rather, schools above 40 percent poverty had students who did not demonstrate further decreases in reading scores, reflecting a floor to the average reading scale score points at elementary schools in FCPS. It is also important to note that this impact was for all students attending FCPS schools, meaning that both students living in poverty and those not from impoverished backgrounds at the same school demonstrate similar declines in their reading performance when attending schools above the 20 percent poverty tipping point.*

Statistical Analyses

This study quantified the overall correlation between school-level poverty and reading pass rates¹ on the SOL tests as medium-sized² and negative (Kendall's $Tau = -.579, p < .001$; see Figure 1 for graphical representation of this relation). That is, schools with higher-poverty levels were likely to have lower rates of students passing the reading SOL and, conversely, schools with lower-poverty levels were likely to have higher rates of students passing. This initially quantified association, however, did not capture the unique impact of school poverty on reading achievement because it intertwines individual student poverty with school poverty. Thus, more sophisticated analyses were used that could view school poverty in isolation and, in particular, consider the relations between school poverty and student reading achievement at the identified tipping points. That is, what was different at schools above and below the poverty tipping points in how students performed? These analyses also allowed the study to quantify to what extent overall school poverty levels were related to a student's reading achievement separately from individual poverty.

¹ Only reading performance was considered in analyzing this question because tipping points in math were inconsistent.

² Effect size (ES) statistics quantify the strength of a phenomenon and, thus, reflect the importance of the relation. An effect size of 0.8 or higher is typically considered a "large" effect, around 0.5 a "medium" effect, 0.2 to 0.3 a "small" effect, and 0 to .1 not meaningful. ES is distinguished from a statistical significance test in that ES estimates the strength of an association (e.g., between school poverty and school achievement), rather than simply assigning a significance level reflecting whether the association could be due to chance. See Jacob Cohen (1988). *Statistical Power Analysis for the Behavioral Sciences* (second ed.).

Multilevel analyses separated the contributions of student-level and school-level poverty on student reading performance. Overall, approximately 10 percent of differences in students' reading scores could be explained by school poverty separate from the poverty background of individual students. School poverty in FCPS was associated with an average decrease of 8 to 18 scale score points on the SOL reading test. Additionally, students attending schools with 20 percent or higher poverty scored approximately seven points lower, on average, than students attending lower-poverty schools (poverty below 20 percent). So, for example, this tipping point analysis indicated that students at the midpoint of Zone I (i.e., school with 10 percent poverty) would score, on average, three points lower than a school with no poverty, while for schools at the midpoint of Zones II (32.5 percent poverty) or III (62.5 percent poverty), all students at these schools would score, on average, 17 and 26 scale score points lower on the reading SOL. Again, all of these decreases are over and above any impacts from students' individual poverty. Thus, this tipping point indicates that schools above the 20 percent poverty rate were not as successful as those below the 20 percent poverty rate even after controlling for student poverty.

The other two tipping points found in the data (at 40 and 45 percent poverty) appear to represent the same pattern. These two points were associated with the reverse influence on student achievement than was found for 20 percent and higher poverty schools. Specifically, students at schools with higher than 40 percent poverty earned 13 to 16 points more than the downward trend would have expected. This does not mean that students at these schools were typically outperforming students at less impoverished schools. Rather, it indicates that students in schools with 40 percent or higher poverty did not demonstrate further decreases in reading scores. Or, stated another way, as poverty levels increased in FCPS schools, students performed worse than peers attending non-impoverished schools only to a point. [For additional details on the regression and hierarchical linear modeling analyses supporting the findings for this question, see Appendix C.]

What Can FCPS Conclude From This Study?

Substantial research has been done showing the significant impact poverty has on student achievement and educational attainment both for individual students and for impoverished schools. Poverty has long been an obstacle to academic success and is often viewed as an insurmountable challenge. It seems logical that as burdens and challenges increase (as is the case with educating impoverished students), the amount of work or effort required to attain a goal also increases. Reform strategies, including various funding formulas, have been tried, and some schools emerge above the challenge. However, it is more typical for many of the same schools to struggle over time to meet school-level performance expectations. Understanding that there is probably no single or guaranteed solution to fully address this challenge, more aggressive approaches should be considered where this challenge has been persistent in spite of tremendous investments and efforts to mitigate it.

This study found a demonstrable and specific link between *overall* (i.e., concentration) school poverty and student achievement. Moreover, this link is in addition to the link between *individual* student poverty and achievement. School poverty was shown to explain about ten percent of differences in elementary reading scores across the school division. In research, this amount of explanation is considered substantial for a single variable. While this study examined only one year of data due to time constraints, the overall patterns of school-level poverty and student achievement at schools is relatively stable from year to year, so the results should be similar in other years. Furthermore, the findings of this study are similar to those found in an analysis of Montgomery County data, which also found poverty "tipped" schools when it reached 20 percent (Schwartz, 2010).

Currently, in FCPS, the range of poverty in elementary schools is 0 to 80 percent; the average poverty rate for elementary schools is approximately 26 percent. During SY 2011-12, 66 elementary schools in FCPS had school poverty rates below the 20 percent tipping point, while 38 schools fell between 20 percent and 39 percent, and 34 schools had poverty rates above 40 percent. The tipping point evidence from this study indicates that the same student attending one of the 66 FCPS elementary schools with poverty levels below 20 percent would have higher reading achievement than that same student attending an FCPS school with poverty that rises above 20 percent. These differences probably arise due to the challenges placed on staff instructing substantial numbers of students living in poverty (who often have unmet basic needs (e.g., food, clothing, shelter) and less background knowledge, among other typical concerns), as well as the collection of differences found in higher poverty schools (such as lower rates of teacher retention, greater numbers of novice staff, and lower levels of parent involvement).

For some schools, this tipping point of overall school poverty means the difference between being viewed as successful or not, which highlights the potential importance of controlling school poverty levels as much as possible. This study also found a slowing of the downward achievement trend once school poverty level reached 40 percent. This finding shows that while student achievement is still negatively impacted by 40 percent or more overall poverty, there is a floor to how low students' average SOL scores fall. These statistically significant findings may indicate the positive influence of the infusion of additional resources into these schools through Title I.

What Actions Might FCPS Consider Taking Based on This Study?

As demonstrated in this study, there is a linear relationship between poverty and achievement. The greater the poverty, the less overall achievement observed at the typical school. Moreover, this study showed that when schools exceed 20 percent poverty, there is a dramatic drop in reading achievement. This drop tends to level off when school poverty reaches somewhere between 40 to 45 percent, meaning that performance does not continue to drop steeply, but it remains lower than schools below the 20 percent poverty level. The study also shows that FCPS has a few schools that have been successful in meeting their performance expectations (percent passing reading SOLs) despite their high levels of poverty. However, these schools appear to be outliers from the general patterns of performance.

This paper was initiated based on the concern for the demand that school poverty places on higher poverty schools as they seek to educate all children at high levels. Based on the findings, two general areas of action seem appropriate for FCPS to consider:

Area 1 – Reducing Poverty at FCPS Elementary Schools: While 20 percent poverty was the tipping point identified in this study for optimal school benefit to students, it is not likely a feasible target for FCPS (where school poverty averages 26 percent) and at most other large school divisions (which typically have even higher average school poverty levels). However, the linear relationship between school poverty and achievement suggests that any reductions in poverty at schools above the tipping point are likely to be beneficial, especially in combination with the other best practices that have been identified by FCPS. The actions suggested here are based on the notion that reducing the level of school poverty at a school will lessen the challenge faced by staff, better positioning them to fully engage students in all educational opportunities at the school, as well as to implement optimal best practices for providing such opportunities. FCPS could consider reducing the level of poverty of schools that have demonstrated persistent achievement challenges despite other efforts. More specifically, the following is a list of potential opportunities for considering reductions in school poverty:

- New schools: Assigning students to new schools may be considered towards the goal of balancing or minimizing the level of overall school poverty as much as reasonably possible at the new school and nearby schools.
- Special academic programs at school sites: Higher poverty schools may be considered as host sites for programs that traditionally attract higher socio-economic populations to draw voluntarily a broader economic population of students.
- Under- or over-filled schools: When student membership at schools considerably exceeds or falls short of expected levels, explore the opportunity for moving students with the goal of maximizing the number of schools with poverty levels below 20 percent.
- New neighborhood construction: Work with county agencies that influence socio-economic integration of neighborhoods to create natural distributions of socio-economic levels.

Area 2 – Maximizing School Conditions in Higher Poverty FCPS Schools: In situations where the level of poverty cannot be reduced at schools, FCPS could consider whether it has maximized higher poverty schools’ capacity to engage and instruct their students. That is, while FCPS has engaged in many creative and research-based practices, there are areas that can still be explored to increase the capacity of schools with 20 percent or more poverty to meet the needs of their students. Based on a recent conversation with the FCPS Leadership Team, the following is a list of opportunities that may be explored:

- Teacher quality: Ensure that higher poverty schools have equally experienced teachers and as stable a teaching force as the rest of the division. This would include the recruitment and retention of highly experienced and committed teachers.
- Leadership quality: Ensure that higher poverty schools have equally experienced principals and assistant principals as the rest of the division. These principals should be able to leverage changes in the division that impact their schools’ success.
- Best Practices: Ensure that all schools have systematic and ongoing access to successful practices based on the experiences of other FCPS schools or research.
- Resources: Ensure that higher poverty schools understand how to access resources, including central office staffs, and consistently make best use of all resources provided.
- Parent and Community Engagement: Ensure that higher poverty schools understand how to build effective family and community connections.

Stakeholders and decision makers should be engaged in conversations about the opportunities for action related to the tipping point findings. Such discussions could solicit opinions about how else this information could be considered, including the opportunities described for Areas 1 and 2 above, as well as communications, resources, and other policy and funding issues.

Appendix A Importance of School Poverty

Reports have long noted important differences between the achievement of students in high- and low-poverty schools.

About 91 percent of 12th graders in low-poverty schools graduated in comparison to 68 percent of 12th graders in high-poverty schools. Since 2000, the average percentage of graduates in high-poverty schools has declined by 18 percent. (Condition of Education, 2010).

About 28 percent of high school graduates from high-poverty schools attended four-year colleges after graduation, compared with 52 percent of high school graduates from low-poverty schools (Condition of Education, 2010).

Educators and the public commonly attribute these findings to the poverty of individual students. However, some reports more clearly tease apart the impact of school poverty; that is, the concentration of poverty at a school from individual student poverty.

Students in grades 3 to 8 from low-income families scored 16 percentage points higher on reading tests in 2010 if they attended a low-poverty school rather than a high-poverty school. This was intensified for students from high-income families, where those who attended a low-poverty school and scored 31 percentage points higher than those who attended a high-poverty school (Gonen, 2011).

Coleman et al. (1966, p. 22) found, “Children from a given family background, when put in schools of different social composition, will achieve at quite different levels.” In addition, a study by Kennedy and colleagues found:

. . . the relationship between family poverty status and student achievement is not as strong as the relationship between school poverty concentrations and school achievement averages. Non-poor students attending schools with high concentrations of poor students were found to be more likely to fall behind than poor students who attend schools with small proportions of poor students. (1986, p. 22)

In a *Research Watch* report over a decade ago, a large suburban school division noted the following:

A large body of research shows that an individual student who is eligible for free or reduced price lunch is at risk for academic failure. The risk factors for individual students can be ameliorated by extra support and academic assistance to ensure academic success. A high concentration of low-income students in a school, however, appears to have negative effects on students, teachers, and the school, and these effects extend beyond the effect of the individual student’s economic condition. (Research Watch Report 99.20, p. 1, Wake County Public Schools)

These and similar findings suggest that academic success is influenced not only by the poverty level of an individual child, but also by the concentration of poor children at the same single school. Since the research shows that these same poor children tend to learn well in schools with low percentages of poor children, it is clear they have the ability to learn.

FCPS has consistently exerted the greatest commitment and effort each day toward educating all of its students. Like many other high-performing school divisions, FCPS has specific unmet challenges regarding the education of all of its students. During the 2012-13 school year, FCPS participated in and received results from the Organization for Economic Co-operation and Development's (OECD) *Test for Schools (Based on PISA) Pilot Trial*. This test provided research and results previously unexplored by FCPS. One of the report sections, *Your School's Results Compared with Public and Private Schools in the United States*, included the general finding:

. . . in comparing the difference in performance explained by students' socio-economic background with the variance among schools' socio-economic background in the United States, the variance explained by schools' socio-economic background is almost 8 times greater than the variance explained by students' socio-economic background. . . . This indicates that students attending the same school do not [or may not] display different abilities or effort, but that the way in which students are allocated to schools in the United States results in large gaps and marked variations in performance in schools. (OECD Test, Langley High School, 2013, p. 84)

In lay terms, the OECD's finding means that low-income students who attend low-poverty schools typically outperform low-income students who attend high-poverty schools. The finding from OECD indicated the importance of attending to overall school poverty levels separately from individual student poverty.

Appendix B

Tipping Point Studies Outside of the Education Field

What is a Tipping Point?

The concept of a “tipping point” generally refers to a critical point when unprecedented changes occur rapidly with irreversible effect (Bhatanacharoen, Greatbatch, and Clark, 2011). This term was more explicitly defined when used in the physical sciences, specifically, in the context of climate change. Lenton and colleagues (2008) defined a tipping point as “a critical threshold at which a tiny perturbation can qualitatively alter the state of development of a system.” In Gladwell’s book, *The Tipping Point: How Little Things Can Make a Big Difference* (2000), he defined tipping point as “the moment of critical mass, the threshold, the boiling point” (p. 12) that once reached is hard to stop, and it becomes an epidemic or social movement. Though the concept of tipping points is used widely throughout different disciplines and in different contexts, the basic concept holds true; a tipping point is a threshold at which an inevitable change will occur.

Tipping Point Studies in Education

Within the education field, there are few published works that address the relation between poverty and achievement with the consideration of a possible tipping point or threshold effect. The OPE identified two studies and, while only one specifically mentioned a tipping point, the other provided evidence supporting the concept.

The Wake County Public School System (WCPSS) Evaluation and Research Department published three reports from 1999-2004 about the impact of school poverty. The culmination of these reports pointed to the importance of considering the poverty distribution within a school. WCPSS found that when their schools’ percentage of low-income students exceeded 40 percent, student performance dropped significantly. Using achievement growth data and free and reduced lunch eligibility as an indicator for poverty, researchers found that high-poverty schools experienced greater challenges in sustaining the growth shown in previous years. In this study, researchers also found that school poverty had a small and statistically significant negative effect on academic growth, with the magnitude varying across subjects and grade level. WCPSS concluded that small reductions in the concentration of poverty were unlikely to have meaningful effects, but that larger reductions would produce positive effects in student achievement growth that would be both statistically and educationally significant (Banks, 2001). To this end, WCPSS implemented a policy where no more than 40 percent of students at any given school should be receiving free or reduced lunch.

In Montgomery County Public Schools (MCPS), a study on the impact of school poverty on student achievement was conducted that also provided evidence for the existence of a tipping point. Utilizing data on students whose families used Montgomery County’s inclusionary housing program as an indicator of low income, Swartz (2010) found that the aggregated income level of the school had a significant impact on student achievement. After two years in MCPS, students in public housing performed equally on standardized math tests regardless of the poverty level of their schools; however, by the fifth year, a statistical difference emerged, where those students in low-poverty schools were now outperforming those in moderate-poverty schools. By the seventh year, the students in low-poverty schools performed an average of eight points higher than children in moderate-poverty schools. Further analysis revealed that the students in the lowest-poverty schools were catching up to their average non-poor district peers. The differences found between students in public housing in math

performance was never found to be statistically significant in reading, though the low-poverty students did still score five points higher on average.

Swartz (2010) then found that students living in public housing and attending schools with 20 percent or less poverty outperformed public housing students who attended schools with more than 20 percent in reading. For math, Swartz found that the students attending schools with a poverty level below 35 percent outperformed students at schools with a poverty level above 35 percent. In this study, the researcher provided evidence for the existence of a tipping point; but, there was no particular plan of action taken. This lack of action was also due to the fact that Montgomery County already has a means to ensure economic integration through their housing policy.

Tipping Point Studies in the Social Sciences

Giles, Cataldo, and Gatlin (1975) researched the phenomenon of “white flight” in schools after the establishment of court-ordered desegregation and busing. Gatlin et al. (1975) found that the tipping point for an integrated school to become a majority black school was 30 percent black student enrollment in some parts of the country.

Card, Mas, and Rothstein (2008) used census tract data from 1970 through 2000 to examine a possible tipping point in the relationship between minority presence in a neighborhood and a reduction in the white population. They found strong evidence that a tipping point in neighborhood occupancy exists in most cities, with tipping points ranging from 5 percent to more than 20 percent, so that once minority share in a neighborhood exceeds the tipping point, there is a rapid change where all of the white neighbors leave (Card et al., 2008).

Tipping points also have been studied to understand the nonlinear relationship between poverty and crime. In 2011, Hipp and Yates conducted analysis on crime data from 25 cities and found a diminishing threshold effect for poverty and crime, such that crime did increase with increasing levels of poverty until the level of poverty reached a threshold. This analysis showed that contrary to what others have hypothesized, crime does not increase exponentially with the increase of poverty. Crime increases until neighborhood poverty reaches approximately 20 percent to 40 percent poverty, after which point, crime levels off.

Tipping Point Studies in the Environmental Sciences

The tipping point concept has also been used in the environmental sciences. Previous research supported the concept that inbreeding led to the increased risk of extinction; however, Frankham (1995) used the tipping point concept to study data sets with information on inbreeding and extinction. This study revealed a threshold effect: once inbreeding reached a particular point or tipping point, the extinction rate sharply increased in all of the animals included in the study. The finding has helped conservation scientists to guard against extinction related to inbreeding among animals.

Kim, Lee, Hong, Ahn, and Kim (2004) conceptualized a tipping point study to understand a critical relationship between the ozone and mortality rates in Seoul, Korea. (Kim et al.) The ozone is an unstable, poisonous allotrope of oxygen, O₃, which is formed naturally in the ozone layer. The 2004 study found that the tipping point exists between 20 and 30 parts per billion (ppb) in the air for all seasons, but specifically in the summer, the tipping point was around 40 ppb. When this threshold or tipping point in the ozone level was reached, mortality rates increased significantly.

Enton et al. (2008) applied the concept of tipping points in climatology by creating “tipping elements” to describe subsystems of the earth that can be qualitatively impacted by small disturbances. An example of a tipping element is the Greenland Ice Sheet (GIS), which is shrinking due to summertime temperature increase. Researchers predict that the tipping point is any temperature greater than or equal to three degrees Celsius. Once this tipping point is reached, researchers predict melting at too rapid a rate to be accommodated by the current sea level without endangering the earth’s population.

Tipping Point Studies in the Medical Sciences

Tipping points have also been useful concepts in medical research. Mainous and Hueston (1994) used a threshold effect to explain the relationship between mothers’ exposure to passive smoke and birth weights. Mainous and Hueston (1994) found no significant difference between mothers exposed to passive smoke and those who reported no exposure to smoke in their childbirth rates. However, when exposure to smoke was ordered into categories of exposure that ranged from very low to high, there was a significant difference between high exposure and the other categories (Mainous & Hueston, 1994).

Harrington et al. (2010) used the tipping point concept to find a critical period for preventing childhood obesity. Using medical records for individuals with a body mass index at or above the 85th percentile, researchers found that it was the first two years of life that appear to have set these individuals on a pathway to obesity. That is, approximately half of the overweight people became overweight at less than two years of age and 90 percent at less than five years of age. These data identify a tipping point for the greatest intervention since beyond this point; obesity appears to be the most likely course of development.

Cady, Michaelson, and Chung (2011) explored the tipping point related to breast cancer mortality rates. The breast cancer mortality rate has declined over the past decades, and the most significant decline was witnessed between 1986 and 1991. The researchers determined if they could find malignant tumors (through mammograms) within a specific size range, the female has a better chance of survival. Of course, the smaller the malignant tumor, the better the chances of survival, but the researchers found the tipping point for survival which is about 2.0 cm. Over time, this became the average size associated with breast cancer for survival.

Methodologies

Determining tipping points involves the use of statistical procedures that ensure reliable results. However, the procedures can vary since there are different ways to ask questions related to tipping points, different types of data, and many statistical procedures to answer the questions. For example, one might ask if there is a single tipping point (independent variable) associated with a single (dependent) value, or one might ask if there is a range of tipping points associated with a range of values. For example, is 50 percent school poverty associated with a school’s designation as “unsuccessful” more than (or equal to) any other percentage of school poverty?

Literature does not indicate any apparent methodological trends between disciplines or over time periods in the analysis of tipping points. Some studies have used a very basic technique of simply graphing data and detecting any noticeable break that might indicate where a tipping point seemed evident (Giles, Cataldo & Gatlin, 1975; Cady, Michaelson, and Chung, 2011). However, the most common analysis used for these kinds of questions is correlational in nature, typically regression. Regression analysis is defined as “any of several statistical techniques concerned with predicting some variable by knowing others.” Regression is used to answer such questions as “how well can I predict the

values of one variable, such as annual income, by knowing the values of another variable, such as level of education” (Dictionary of Statistics and Methodology, 1999). Within regression analysis, there are various approaches used to study tipping points, such as linear, cubic, multilevel, etc. Each type of regression analysis works best for understanding a specific type of relation between two or more variables. So, if one type of data does not fit an approach, the researcher would try another approach to find the best fitting model. The important point is that researchers have been required to determine the best regression model for their data where a tipping point is expected.

Appendix C Additional Details on Analyses

Data and Sample:

The school-level SY 2011-12 data used in this study (percentage of students receiving free or reduced fee meals, percentage of students passing math and reading SOLs) were drawn from Virginia Department of Education data. This resulted in a sample of 138 elementary schools. Individual student data collected for all students in these 138 elementary schools from SY 2011-12 (free or reduced fee meals status, math SOL score, reading SOL score) came from FCPS testing and student information data. All students in grades with reading and mathematics tests at these schools (grades 3 through 6) were included in analyses. This resulted in inclusion of 42,226 students for math and 39,801 students for reading in 138 schools.

School-Level Analyses

Measures and Analysis Plan:

In total, we employed only three measures to investigate school-level poverty tipping points. First, to measure the amount of poverty present in schools, the percentage of students who received free and reduced meals in Fall 2011 was used. Second, our two dependent measures were the percentage of students passing the reading and math SOL exams in SY 2011-12.

Our first step was to examine the data in order to learn if there was a relation between school poverty and student achievement. We first tested our three measures for normality and homogeneity of variance. We performed correlational analyses to understand whether a significant linear relation existed between the two factors. Finally, we computed transformed variables and included them in regression analyses in an attempt to capture any nonlinear trends that would reflect a tipping point.

Normality:

Given that our sample sizes for all three measures were 138 cases, we performed the *Shapiro-Wilk* test to determine whether these three measures were normally distributed. As can be seen in Table C-1, our school poverty measure and both student achievement measures had non-normal distributions ($p < .001$). This finding influenced what statistic we would use when running correlational analyses, since it indicated nonparametric analysis would be most appropriate.

Table C-1: Shapiro-Wilk Tests for Normality

	Shapiro-Wilk Test	Df	Sig
Free and Reduced Price Meals in 2012 (%)	.912	138	***
Passing Reading in 2011-12 (%)	.931	138	***
Passing Math in 2011-12 (%)	.964	138	***

* $p < .05$; ** $p < .01$; *** $p < .001$

Bivariate Correlations:

Using correlations, this study examined the relation between the school poverty level and the proportions of students passing their reading and math SOLs. Correlations measure the linear

relationship between two variables. The most common correlation measure used is a Pearson's r ; however, Pearson's r assumes that both variables are distributed normally (Field, 2005). Since all three of our measures were non-normally distributed, we measured correlations by calculating the Kendall's τ statistic which rank orders the variables and examines the concordance and discordance among these rankings. Kendall's τ also has several benefits over other correlation measures, such as Spearman's r , including better measurement for small datasets and improved estimates of the population (Gibbons, 1993). Values for correlations can range between +1.00 and -1.00, with values close to zero representing no relationship, and values close to positive or negative 1 indicating perfect correspondence of one score with another score.

Table C-2 presents the correlations between school poverty and student achievement in reading and math. As can be seen below, both relations were statistically significant. Indeed, schools with higher poverty rates were significantly more likely to also have lower rates of students passing reading ($r=-.579$) and math ($r=-.525$).

Table C-2: Correlations between School Poverty and Student Achievement (n=138)		
Student Achievement	Kendall's τ	Significance level
Passing Reading in 2011-12 (%)	-.579	***
Passing Math in 2011-12 (%)	-.525	***

* $p < .05$; ** $p < .01$; *** $p < .001$

Figure C-1 is a graph of student-level achievement in reading as a function of school-level poverty. Each diamond in the figure represents one FCPS elementary school and positions it in the graph with respect to level of school poverty (horizontal axis) and pass rates on the reading SOL. Figure C-2 is similar to Figure C-1 but shows student-level achievement in math as a function of school-level poverty. Both figures depict the linear relation between the two measures (which was quantified by the significant Kendall's τ values) and is represented graphically on these figures with the sloping black line.

Figure C-1: School Poverty and Student Achievement in Reading

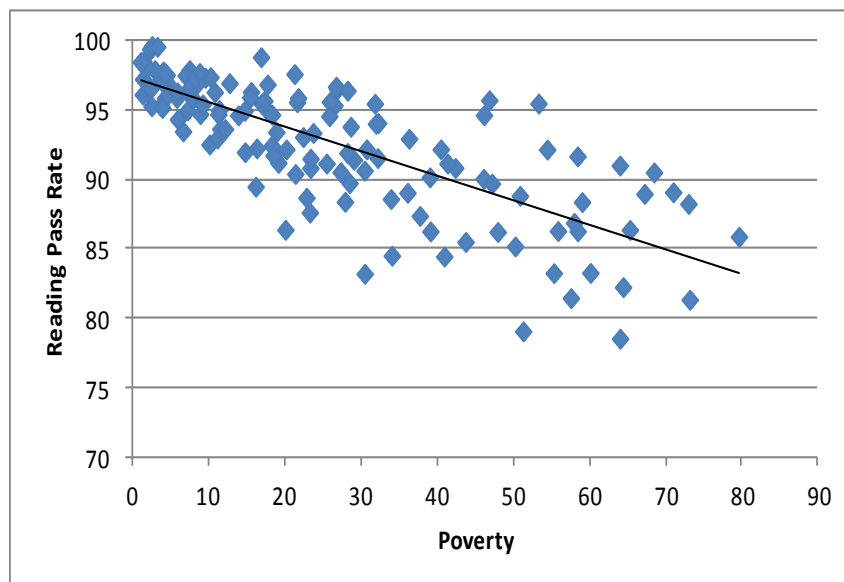
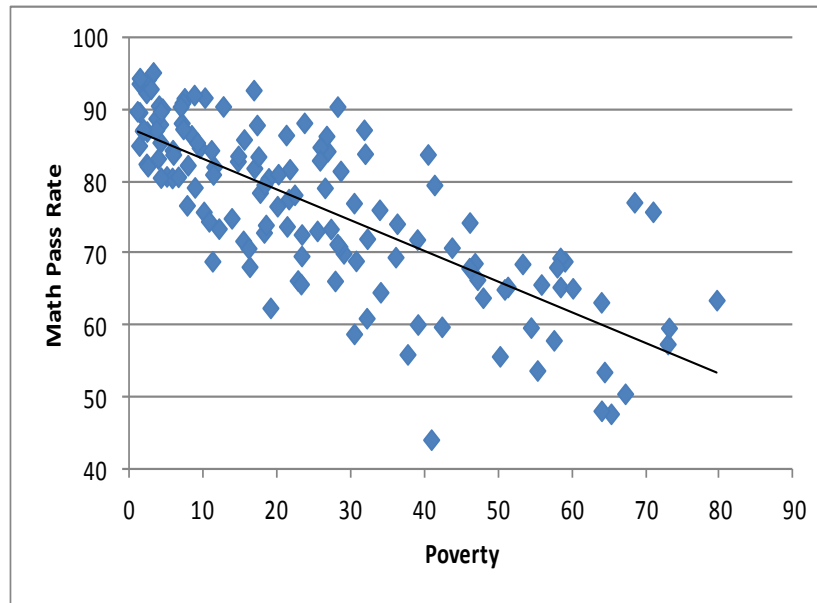


Figure C-2: School Poverty and Student Achievement in Math



Regressions with Transformed Variables

Once we established a statistically significant relation between poverty and student achievement, the next step in our analyses was to examine where the tipping point occurred for both reading and math. In this study, we had hypothesized that there was a tipping point at which a fixed amount of action in one variable (i.e., five percent reduction in poverty) has maximal impact on a second variable (i.e., student achievement). Finally, we attempted to use what we had learned to create quantitative measures of tipping points which can be examined using additional multilevel models.

The nonlinearity in the relation between school poverty and student achievement can be conceptualized using transformed, nonlinear terms. Table C-3 presents transformations used to capture nonlinear data. An Ordinary Least Squares (OLS) regression was run for both a reading and a math model using the main effect of school poverty and each of the transformations. As can be seen below, the main effect of school poverty was statistically significant across all models. Additionally, both of the nonlinear transformations (squared, cubed) were significant. These findings suggest that over and above the negative linear trend relationship (statistical evidence that school poverty is related to lower student achievement), there is a nonlinear relationship that would signify a tipping point.

Table C-3: Significance Levels of OLS Regressions with the Main Effect of Poverty and Different Transformation Options

Transformations	Reading Achievement Significance Level		Math Achievement Significance Level	
	Main Measure	Nonlinear Measure	Main Measure	Nonlinear Measure
Squared (X^2)	***	*	***	*
Cubed (X^3)	***	*	***	*

*p<.05; **p<.01; ***p<.001

Student- and School-Level Analyses

Measures

The multi-level analyses used outcome, school-level, and student-level measures, described more fully below:

- **Outcome Measures:** Student achievement was measured using student SOL test scores in math and reading in 2012.
- **School-Level Measures:** In total, we used two school-level measures. First, we included a schoolwide poverty measure (percentage of students who received free and reduced price lunches). Our second set of measures explored the additional effects that attending a high-poverty school may have upon student test scores. Because we did not know where on the spectrum of school poverty a “high-poverty” school should be defined, we conducted a number of analyses, operationally defining “high poverty” at 5 percent intervals with lower thresholds between 20 percent and 60 percent. This resulted in nine analyses, with the most inclusive definition of high poverty to include those schools with between 20 percent and 100 percent. At the other end of the range, high poverty would include all schools with between 60 percent and 100 percent poverty.
- **Student-Level Measures:** This study included student poverty status as measured by whether they received free and reduced price lunches.

Descriptive Statistics

Table C-4 presents descriptive statistics for the measures used in this study broken out by the math and reading samples.

Table C-4: Means and Standard Deviations for School and Student Characteristics in Math and Reading		
	Math Mean (sd)	Reading Mean (sd)
Sample Sizes		
Student-Level	42,226	39,801
School-Level	138	138
Student-Level		
Free/Reduced Lunches	27% (45%)	25% (43%)
Achievement 2012	-.00 (62.85)	.00 (66.77)
School-Level		
Poverty Rate	26% (20%)	26% (20%)
Poverty 20-100%	52% (50%)	52% (50%)
Poverty 25-100%	43% (50%)	43% (50%)
Poverty 30-100%	34% (48%)	34% (48%)
Poverty 35-100%	28% (45%)	28% (45%)
Poverty 40-100%	24% (43%)	24% (43%)
Poverty 45-100%	20% (40%)	20% (40%)
Poverty 50-100%	17% (37%)	17% (37%)
Poverty 55-100%	13% (34%)	13% (34%)
Poverty 60-100%	8% (27%)	8% (27%)

Hierarchical Linear Models (HLM)

This section brings together the information from the student-level and school-level datasets in multilevel models designed to assess the different poverty factors that influenced a student's achievement in math and reading.

Unconditional Model:

The first step in a multivariate analysis is to estimate the unconditional or null model. For this study, the unconditional model was estimated in order to determine how much between-level variance existed in student achievement in mathematics and reading. In the following equation, β_{0j} represents the average level student achievement across schools and e_{ij} represents the variance of student achievement between teams.

$$Y (\text{Student Achievement})_{ij} = \beta_{0j} + e_{ij} + u_{0j}, \text{ where } u_{0j} \sim N(0, \tau_{00})$$

The results of the unconditional models indicated that the average student math and reading scores of students included in the study were -2.7 and -3.5 points below the average student test scores in these subject areas within FCPS. Additionally, the null models indicated that there was significant between-level variation in student achievement at the school level (Math U_{0k} $p=.001$; Reading U_{0k} $p<.001$). These significant error terms suggest that it was necessary to control the between school variation in student achievement, meaning that some schools had significantly different levels of variance in the dependent variable of student achievement.

Multicollinearity and Model Selection:

Before we selected our final models, variables in our analyses were assessed for multicollinearity using bivariate correlations and variance inflation factors (VIFs). In 1988, Cohen suggested that bivariate correlations higher than .50 (or lower than -.50) are large. Among our variables, we did not discover any large correlations, although the student- and school-level poverty measures were more strongly related than our other measures. VIF is a ratio of coefficients that assesses the predictability of an independent variable by another independent variable. In our model, no predictor had a VIF score over the threshold of 4, which is well below the recommended cutoff point of 10 for VIF scores (Neter, Kutner, Nachtsiem, & Wasserman, 1996).

Math Achievement Models:

Across all of the math achievement models, the student poverty measure was a strong and consistent predictor of math exam scores. In Table C-5, students' poverty status accounted for almost a 40-point decrease in math exam scores.

Unfortunately, the school-level poverty rate was not a consistent predictor of math exam scores. In Table C-5, school poverty was not a significant predictor of student math achievement in two of the models. When it was significant, school poverty explained between .3 and .5 points per percentage of school poverty. On average, schools in Fairfax County had a 26 percent school poverty rate, which translates into 8 to 13 points lower.

Finally, there was one significant high school poverty tipping point measure. Students who attended schools in which 45 percent or more of the students received free and reduced price meals were significantly more likely to have test scores that were approximately 12 points higher than would have

been predicted by the downward linear trend. Given that this was the only statistically significant high-poverty measure finding, this finding may be due solely to chance.

Reading Achievement Models:

Across all of the reading achievement models, the student poverty measure was a strong and consistent predictor of student achievement in reading exam scores. In Table C-6, students' poverty status accounted for a 44 point decrease in their exam scores.

In Table C-6, the school-level poverty rate was a consistent predictor of student achievement. For every extra percentage point of school poverty, students' reading exam scores fell between .3 and .7 points. With the average school poverty rate at 26 percent, this means that on average, a student's school poverty rate translated into a decrease in reading exam points of 8 to 18 points.

There was evidence of multiple tipping points in the reading models. Indeed, for students who attended a school in which 20 percent or more students received free and reduced lunches, these students scored 6-7 points lower than those attending a school with less than 20 percent poverty over and above the downward trend line. It is also interesting to note that students who attended schools with significantly more poverty (+40 percent, +45 percent, +55 percent, and +60 percent) were significantly more likely to receive slightly better test scores than would be expected based on the downward trend line. Although this finding is interesting, it is likely that these statistically significant findings indicate a floor to the school poverty finding, meaning that students in these more impoverished schools performed worse than their peers up until a point.

Table C-5: Multilevel Models Examining School Poverty Tipping Point for Student Achievement in Math, SY 2011-12
(Values highlighted in yellow identify statistically significant tipping points.)

	Model 1: Main	Model 2: 20%	Model 3: 25%	Model 4: 30%	Model 5: 35%	Model 6: 40%	Model 7: 45%	Model 8: 50%	Model 9: 55%	Model 10: 60%
Student-Level	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)
Poverty 2012	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***	-39.6 (.7) ***
Intercept	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)	-1.5 (1.3)
School-Level										
Poverty Rate 2012	-.3 (.1) ***	-.2 (.1)	-.4 (.1) **	-.2 (.1)	-.3 (.1) *	-.4 (.1) ***	-.5 (.1) ***	-.4 (.1) ***	-.4 (.1) ***	-.3 (.1) ***
Poverty 20-100%		-5.6 (4.3)								
Poverty 25-100%			3.5 (4.7)							
Poverty 30-100%				-2.3 (5.2)						
Poverty 35-100%					2.1 (5.7)					
Poverty 40-100%						7.1 (5.7)				
Poverty 45-100%							11.6 (5.7) *			
Poverty 50-100%								7.8 (5.7)		
Poverty 55-100%									8.6 (5.7)	
Poverty 60-100%										4.9 (.6.1)
Variance Components										
PEV Student-Level	6.4%	6.4%	6.4%	6.4%	6.4%	6.4%	6.4%	6.4%	6.4%	6.4%
PEV School-Level	51.1%	51.4%	50.9%	50.8%	50.8%	51.3%	52.3%	51.4%	51.6%	50.9%

*p<.05; **p<.01; ***p<.001

Table C-6: Multilevel Models Examining School Poverty Tipping Point for Student Achievement in Reading, SY 2011-12
(Values highlighted in yellow identify statistically significant tipping points.)

	Model 1: Main	Model 2: 20%	Model 3: 25%	Model 4: 30%	Model 5: 35%	Model 6: 40%	Model 7: 45%	Model 8: 50%	Model 9: 55%	Model 10: 60%
Student-Level	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)	B (S.E.)
Poverty 2012	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***	-44.0 (.8) ***
Intercept	-1.9 (1.1)	-1.9 (1.0)	-1.9 (1.1)	-1.9 (1.1)	-1.9 (1.1)	-1.9 (1.0)	-1.9 (1.0)	-1.9 (1.1)	-1.9 (1.1)	-1.9 (1.1)
School-Level										
Poverty Rate 2012	-.5 (.1) ***	-.3 (.1) ***	-.5 (.1) ***	-.5 (.1) ***	-.6 (.1) ***	-.7 (.1) ***	-.7 (.1) ***	-.6 (.1) ***	-.5 (.1) ***	-.5 (.1) ***
Poverty 20-100%		-7.2 (3.5) *								
Poverty 25-100%			1.4 (3.9)							
Poverty 30-100%				-.3 (4.3)						
Poverty 35-100%					7.5 (4.6)					
Poverty 40-100%						12.7 (4.6) **				
Poverty 45-100%							15.7 (4.5) ***			
Poverty 50-100%								6.2 (4.7)		
Poverty 55-100%									3.9 (4.7)	
Poverty 60-100%										5.0 (5.1)
Variance Components										
PEV Student-Level	6.8%	6.8%	6.8%	6.8%	6.8%	6.8%	6.8%	6.8%	6.8%	6.8%
PEV School-Level	73.3%	74.0%	73.1%	73.1%	73.6%	74.6%	75.5%	73.5%	73.3%	73.3%

*p<.05; **p<.01; ***p<.001

References

- Aud, S., Hussar, W., Planty, M., Snyder, T., Bianco, K., Fox, M., Frohlich, L., Kemp, J., Drake, L. (2010). The Condition of Education (2010). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC.
- Banks, K. (2001, March). The effect of school poverty concentration in WCPSS (Report No. 01.21). Wake County Public School System.
- Bhatanacharoen, P., Greatbatch, D., & Clark, T. (2011). The tipping point of the “tipping point” metaphor: Agency and process for waves of change. *Institute of Hazard, Risk and Resilience Blog at Durham University, Durham, England*. Retrieved from <http://www.dur.ac.uk/resources/ihrr/tippingpoints/OLKCPaperforIHRR.pdf>.
- Cady, B., Michaelson, J. S., & Chung, M. A. (2011). The “tipping point” for breast cancer mortality decline has resulted from size reductions due to mammographic screening. *Annals of Surgical Oncology, 18*, 903-906.
- Card, D., Mas, A., & Rothstein, J. (2008). Tipping and the dynamics of segregation. *The Quarterly Journal of Economics, 177-218*.
- Coleman, J. S., Campbell, E.Q., Hobson C.J., McPartland, J., Mood, A., Weinfeld, F.D., & York, R. (1966). *Equality of Educational Opportunity*. Washington, DC: U.S. Government Printing Office.
- Field, A. (2005). *Discovering statistics using SPSS (2nd ed.)*. London: Sage.
- Frankham, R. (1995). Inbreeding and extinction: A threshold effect. *Conservation Biology, 9 (4)*, 792-799.
- Gibbons, J.D., (1993). *Nonparametric statistical methods*. London: Sage Publications.
- Giles, M. W., Cataldo, E. F., & Gatlin, D. S. (1975). White flight and percent black: The tipping point re-examined. *Social Science Quarterly, 56*, 85-92.
- Gladwell, M. (2000). *The tipping point: How little things can make a big difference*. Little, Brown and Company.
- Gonen, Y. (2011, September 14). Schools with high poverty level negatively impacts student performance. *New York Post*. Retrieved from http://www.nypost.com/p/news/local/schools_with_high_poverty_level_kmxSHcserplyC4XqjsFiML.
- Graber, C. R. (2009). Factors that are predictive of student achievement outcomes and an analysis of these factors in high-poverty schools versus low-poverty schools (Doctoral dissertation). Retrieved from <http://udini.proquest.com/view/factors-that-are-predictive-of-pqid:1967908681/>.
- Harrington, J. W., Nguyen, V. Q., Paulson, J. F., Garland, R., Pasquinelli, L. & Lewis, D. (2010). Identifying the “tipping point” age for overweight pediatric patients. *Clinical Pediatrics, 49*, 638-643.
- Kennedy, M. M., Jung, R. K., and Orland, M. E. (1986). Poverty, achievement and the distribution of compensatory education services. An interim report from the National Assessment of Chapter 1, Office of Educational Research and Improvement, U. S. Department of Education. January.

- Kim, S.-Y., Lee, J.-T., Hong, Y.-C., Ahn, K.-J., & Kim, H. (2004). Determining the threshold effect of ozone on daily mortality: An analysis of ozone and mortality in Seoul, Korea, 1995–1999. Elsevier.
- Lenton, T. M., Hermann, H., Kriegler, E., Hall, J. W., Lucht, W., Rahmstorf, S., Schellnhuber, H. J. (2008). Tipping elements in the Earth's climate system. *Proceedings of the National Academy of Sciences of the United States of America*, 105 (6), 1786-1793.
- Mainous, A. G. & Hueston, W. J. (1994). Passive smoke and low birth weight: Evidence of a threshold effect. *Archives of Family Medicine*, 3, 875-878.
- OECD. (2012). How your school compares internationally: OECD test for schools (based on PISA) pilot trial (OECD, Author). Retrieved from http://www.fcps.edu/pla/ost/pisa/school_level_reports/langley_hs.pdf.
- Primer: Education issues - variables affecting student achievement. (nd). Retrieved from Wisconsin Education Association Council website: http://www.weac.org/Issues_Advocacy/Resource_Pages_On_Issues_One/research_materials/primer_variable.aspx.
- Schwartz, H. (2010). Housing policy is school policy: Economically integrative housing promotes academic success in Montgomery County, Maryland. New York: Century Foundation.
- Teacher quality and student achievement: Research review. (2005, November). Retrieved from Center for Public Education website: <http://www.centerforpubliceducation.org/Main-Menu/staffingstudents/Teacher-quality-and-student-achievement-At-a-glance/Teacher-quality-and-student-achievement-Research-review.html>.
- Vogt, W. P. (1999). Dictionary of Statistics & Methodology: A Nontechnical Guide for the Social Sciences (2nd ed.). Sage Publications.